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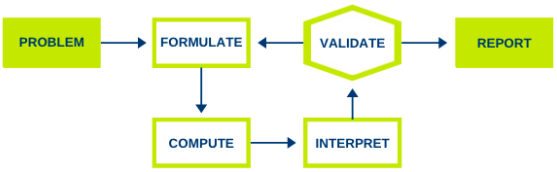
Advice on Modeling

These are some steps that successful modelers often take and questions that they ask themselves. You don't necessarily have to do all of these steps, or do them in order. Only do the parts that you think will help you make progress.



	**Understand the Question** Think about what the question means before you start making a strategy to answer it. Are there words you want to look up? Does the scenario make sense? Is there anything you want to get clearer on before you start? Ask your classmates or teacher if you need to.
	**Refine the Question** If necessary, rewrite the question you are trying to answer so that it is more specific.
	**Estimate a Reasonable Answer** If you don't have enough information to decide what's reasonable, try to come up with an answer that would be too low, and an answer that would be too high.
	**Identify Unknowns** • What are the meaningful quantities in this situation? Write them down. • What information would be useful to know? In order to get that information, you could: look it up, take a measurement, or make an assumption.
	**Gather Information** Write down any of the unknown information that you find. As you work, organize your information in a way that makes sense to you.
	**Experiment!** Try different ideas to make progress toward answering your question. If you are stuck, think about: • Helpful ways to organize the information you have or organize your work • Questions you *can* answer using the information you have • Ways to represent mathematical relationships or sets of data (tables, equations, scatter plots, graphs, statistical plots) • Tools that are available for representing mathematics, both digital and analog
	**Check Your Reasoning** Do you have a first answer to your question? Great! See if it's reasonable. • Make sure you can explain what the answer means in terms of the original problem. • Check your precision: Is your answer overly precise (do you really need all those decimal places)? Not precise enough (were you overly aggressive with your rounding)?
	**Use and Improve Your Model** • Did you make assumptions or measurements? How can you express your model more generally, so that it would work for a range of numbers instead of the specific numbers you used? • What are the limitations of your model? That is, what are some ways it is not realistic? Does it only work for certain inputs but not others? Are there any meaningful inputs affecting the outcome that are not accounted for? If possible, improve your model to take these into account. • What are the implications of your model? That is, what should people or organizations do differently or smarter as a result of what your model shows? What would be effective ways to communicate with them? • What are the areas for further research? That is, what new things are you wondering about that could be investigated, by you or someone else?

How to Interpret the Provided Lift Analysis of a Modeling Prompt

For most mathematical modeling prompts, different versions are provided. Each version is analyzed along five impactful dimensions that vary the demands on the modeler (OECD 2013). Each of the attributes of a modeling problem is scored on a scale from 0–2. A lower score indicates a prompt with a "lighter lift" for students and teachers: students are engaging in less open, less authentic mathematical modeling. A higher score indicates a prompt with a "heavier lift" for students and teachers: students are engaging in more open, more authentic mathematical modeling. This matrix shows the attributes that are part of our analysis of each mathematical modeling prompt. Though not all the attributes have the same impact on what teachers and students do, for the sake of simplicity, they are all weighted the same when they are averaged.

Attribute		DQ Defining the Question	QI Quantities of Interest	SD Source of Data	AD Amount of Data given	M The Model
Lift	Light lift (0)	Well-posed question	Key variables declared	Data are provided	Modeler is given all the information they need and no more	Model is given in the form of a mathematical representation
	Medium lift (1)	Elements of ambiguity; prompt might suggest ways assumptions could be made	Key variables suggested	Modelers are told what measurements to take or data to look up	Some extra information is given and modeler must decide what is important; or, not enough information is given and modeler must ask for it before teacher provides it	Modeler must sift through lots of given information and decide what is important; or, not enough information is given and modeler must make assumptions, look it up, or take measurements
	Heavy lift (2)	Freedom to specify and simplify the prompt; modeler must state assumptions	Key variables not evident	Modelers must decide what measurements to take or data to look up	Modeler must sift through lots of given information and decide what is important; or, not enough information is given and modeler must make assumptions, look it up, or take measurements	Careful thought about quantities and relationships or additional work (like constructing a scatter plot or drawing geometric examples) is required to identify type of model to use

Each version of a mathematical modeling prompt is accompanied by an analysis chart that looks like this sample:

Attribute	DQ	QI	SD	AD	M	Avg
Lift	0	1	0	0	2	0.6

There are other features of a mathematical modeling prompt that could be varied. In the interest of not making things too complex, there are only five dimensions included in the lift analysis. However, a prompt could be additionally modified on one of these dimensions:

- whether the scenario is posed with words, a highly-structured image or video, or real-world artifacts like articles or authentic diagrams
- presenting example for student to explore before they are expected to engage with the prompt, versus the prompt suggesting that the modeler generate examples or expecting the modeler to generate examples on their own
- whether the prompt makes decisions about units of measure or expects the modeler to reconcile units of measure or employ dimensional thinking
- whether a pre-made digital or analog tool is provided, instructions given for using a particular tool, use of a particular tool is suggested, or modelers simply have access to familiar tools but are not prompted to use them
- whether a mathematical representation is given, suggested, or modelers have the freedom to select and create representations of their own choosing

Modeling Rubric[1]

Skill	Score			Notes or Comments
	Proficient	**Developing**	**Needs Revisiting**	
1. Decide What to Model	• Assumptions made are clearly identified and justified. Resulting limitations are stated when appropriate. • Variables of interest are clearly identified and chosen wisely, and appropriate units of measure are used.	• Assumptions are noted but lacking in justification or difficult to find. • Variables of interest are noted, but may lack justification, be difficult to find, or not be measured with appropriate units.	• No assumptions are stated. • No variables are defined.	
	To improve at this skill, you could: • Ask questions about the situation to understand it better • Check the assumptions you're making to see if they're reasonable (Try asking a friend, or imagining that you're a person involved in the scenario. Would those assumptions make sense to you?) • Double-check the variables you've identified: Are there other quantities in the situation that could vary? Is there something you've identified as a variable that is actually fixed or determined? (Remember that more abstract things like time and speed are also quantities.)			
2. Formulate a Mathematical Model	• An appropriate model is chosen and represented clearly. • Diagrams, graphs, etc. are clear and appropriately labeled.	Parts of the model are unclear, incomplete, or contain mistakes.	No model is presented, or the presentation contains significant errors.	
	To improve at this skill, you could: • Check your model more carefully to make sure it really fits well • Consider a wider variety of possible models, to find one that fits the situation better • Think about the situation more deeply before trying to find a model • Convince a skeptic: Pretend that you think your model is inadequate, or ask a friend to pretend to be skeptical of it. What would a skeptic find wrong with your model? Try to fix those things, or explain why they're not actually problems.			

[1] Adapted from IM 9-12 Math Algebra 1 Modeling Prompts https://curriculum.illustrativemathematics.org/HS/teachers/index.html, copyright 2019 by Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license https://creativecommons.org/licenses/by/4.0/.

Skill	Score			Notes or Comments
	Proficient	**Developing**	**Needs Revisiting**	
3. Use Your Model to Reach a Conclusion	● Solution is relevant to the original problem. ● Reader can easily understand the reasoning leading to the solution. ● Relevant details are included like units of measure.	Solution is not well-aligned to the original problem, or aspects of the solution are difficult to understand or incomplete.	No solution is provided.	
	To improve at this skill, you could: ● Double-check your calculations: Show them to someone else to see if they agree, or take a break and look at your calculations again later ● Make sure your calculations are justified by your model: Ask yourself how you decided what to calculate, and see if your reasoning matches up with your model ● Think more deeply about what your conclusions mean in the original scenario: Imagine you're a person involved in the scenario, or explain your conclusions to someone else and see if they have questions			
4. Refine and Share Your Model	● The model's implications are clearly stated. ● The limitations of the model and solution are addressed.	The limitations of the model and solution are addressed but lacking in depth or ignoring key components.	No interpretation of model and solution is provided.	
	To improve at this skill, you could: ● Think more creatively about what your conclusions mean: Ask yourself "If I was involved in this situation, what would I understand better because of these conclusions? What would I want to do next?" ● Be skeptical of your model: What don't you like about it, and what can you do to fix those things? ● Explain your model to someone else: Tell them how it works and why it's good. If you're not sure how it works or why it's good, you might need to change it.			

Modeling Prompt 7A

1. You are planning a vacation. Start by deciding:
    a. Who is going on this trip?
    b. What type of vacation are you taking?
    c. What is your budget? To determine your budget, find the median income for your area and allocate either 5%, 8% or 12% towards your vacation budget.
2. Make a detailed plan outlining the logistics and all expenses for the vacation.
3. Now, analyze your plan. Categorize the different expenses of your trip.
    a. What percentage of your budget are you spending on different categories?
    b. Which of the costs are one-time costs and which costs depend on the length of the vacation, or on the number of people going on vacation?
    c. Describe how the total cost changes if you change the length of the trip or the number of people.
4. Prepare a presentation about your vacation. Include a visual display that would help somebody who wants to take a similar vacation on a different budget decide how long their vacation could be.

Modeling Prompt 7B

You are planning a vacation for a family of four. Your budget for the trip is $3,500. This must cover:

- transportation
- lodging (like hotels or camp sites)
- food
- entertainment

1. Make a detailed plan with the activities and expenses for the trip.
2. Now, analyze your vacation. Categorize the different expenses of your trip.
    a. What percentage of your budget are you spending on different categories?
    b. Which of the costs are one-time costs and which costs depend on the length of the vacation?
    c. What other quantities could be considered variables?
    d. Describe how the total cost changes if you increase the length of the trip.
3. Prepare a presentation about your trip and all expenses. Include what percentages of the total budget are spent in the different categories. Describe mathematically how the cost of the vacation changes if you increase the length of the trip.

Modeling Prompt 8A

1. A charity is going to raise money by having a concert. You are helping to plan the concert. You will need to decide where the concert should be and how much the tickets should cost. You will also need to predict how much profit the charity will make from the concert.

   The charity has done some research about ticket prices. They chose 100 people at random and asked them, “What is the most that you would pay for a ticket to a charity concert?”. The responses are shown in the table.

Ticket price ($)	Number of people who would pay at most this much
10	15
20	22
30	13
40	15
50	17
60	15
65	3

   Here are some questions to guide you as you make your plan:

   - Based on the information from the survey, how much should tickets cost?
   - What kind of performer do you want to have at the concert- a local band? A famous singer? An orchestra? A “battle of the bands” featuring many different groups? More famous performers may draw a larger crowd, but they may also need to be paid more. Do research to find out what a fair amount of money would be.
   - Research some possible concert venues. How much would they cost to rent, and how many people can they hold?
   - If the tickets are sold at the price you recommend, which venue will create the most profit?
   - Should the charity also sell other things at the concert, like food or T-shirts? If you think so, you can also recommend this to the charity and predict how much profit they’ll make.

2. Create a presentation to explain your plan and your reasoning to the charity’s directors. Include an estimate of the costs and profit.

Modeling Prompt 8B

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   The charity has done some research about ticket prices. They chose 100 people at random and asked them, “What is the most that you would pay for a ticket to a charity concert?”. The responses are shown in the table.

Ticket price ($)	Number of people who would pay at most this much
10	15
20	22
30	13
40	15
50	17
60	15
65	3

   You can use this information to find out which ticket price will bring in the most money. Here is how:

   - First, figure out how many people would buy a ticket for $10, how many would buy a ticket for $20, and so on.
   - Then create a graph with the ticket price as the independent variable and the number of people who buy tickets as the dependent variable. Plot the points you found in the first step, and then find a linear equation that shows how the number of people depends on the ticket price.
   - The amount of money that the charity will make from ticket sales is the ticket price multiplied by the number of people who buy tickets. Use the linear equation you found to create a quadratic equation that shows how the price of tickets affects the amount of money the charity will make from the tickets.

   After you have decided on a ticket price, here are some questions to think about:

   - What kind of performer do you want to have at the concert — a local band? A famous singer? An orchestra? A “battle of the bands” featuring many different groups? More famous performers may draw a larger crowd, but they may also need to be paid more. Do research to find out what a fair amount of money would be.
   - Research some possible concert venues. How much would they cost to rent, and how many people can they hold?
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Name: Period: Date:

Student Course Survey

1. How confident did you feel in your own mathematical ability when you began this course?

No confidence	Little confidence	Decent confidence	High confidence

2. How confident are you now in your own mathematical ability, after taking this mathematics course?

No confidence	Little confidence	Decent confidence	High confidence

3. What made you choose the ratings you chose for how you felt at the beginning of the course and now?

Read the following statements and mark the box that corresponds with your thoughts.

	Never	Sometimes	Most of the time	Always
Example: I came to class.				**X**
4. My teacher showed interest in getting to know me.				
5. My teacher cared whether I learned or not.				
6. I felt that it was safe to make mistakes.				
7. My teacher had high expectations for me and my work.				
8. My teacher was prepared to teach me.				
9. I had fun in class.				

Use this space to add your thoughts on any of the comments above.

10. What did your teacher do **best** to support you in this math class?

11. What **improvements** could your teacher have made to support you?

12. What was your **favorite** unit or mathematical concept in math class this year? Why?

13. What mathematical tools helped your learning the **most** in math class this year?

14. Thinking back about your time in this course, when were you having the **most fun** in math class this year?

15. What did you do **best** in math class this year?

16. What do you think you could have done **differently** in math class this year?

17. What are some of your goals for yourself for **future** mathematics courses?

18. Use the space below to give any **additional** thoughts on your experience in this course.

Alphabetical Advice

A

B

C

D

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V

W

X

Y

Z

Adapted from Sarah Carter https://mathequalslove.net/download/alphabetical-advice-pdf/